Spring Block 2

Algebra

Small steps

Step 1	1-step function machines
Step 2	2-step function machines
Step 3	Form expressions
Step 4	Substitution
Step 5	Formulae
Step 6	Form equations
Step 7	Solve 1-step equations
Step 8	Solve 2-step equations

Small steps

Step 9	Find pairs of values
Step 10	Solve problems with two unknowns

1-step function machines

Notes and guidance

In this small step, children begin to formally look at algebra for the first time by exploring function machines. This builds on their work in earlier years using operations and their inverses to find missing numbers.

Children need to learn the meanings of the terms “input”, “output”, “function” and “rule”. At first, they are given a number, told what to do to it using any of the four operations and calculate the output. They then move on to finding the input from a given output, using inverse operations.

Finally, children explore examples where the input and output are given, but the function is not. They should recognise that one rule may fit for some of the numbers given, but not for all, and that they need to find a rule that works for all the numbers.

Things to look out for

- Children may carry out the function on the output when working out the missing input, rather than using the inverse operation.
- Children may find a function that works for some of the numbers given, but not all.

Key questions

- How does the function machine work?
- What is the difference between an input and an output?
- If you know the input and function, how can you work out the output?
- If you know the output and function, how can you work out the input?
- What is the inverse of ______?
- Does your rule work for all the sets of numbers?

Possible sentence stems

- If the input is ______, the output is ______
- If I know the output, I need to …
- If the input is ______ and the output is ______, then the function is ______

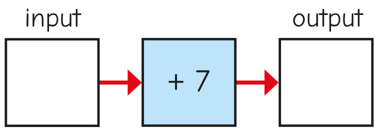
National Curriculum links

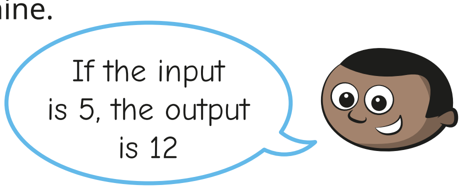
- Use simple formulae
- Generate and describe linear number sequences

1-step function machines

Key learning

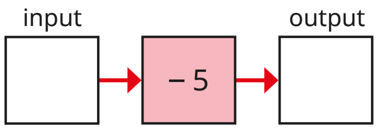
- Mo has made a function machine.





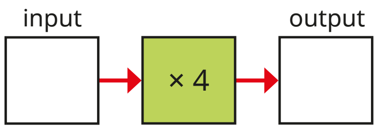
  - If the input is 7, what is the output?
  - If the input is 4,023, what is the output?

- Complete the table for the function machine.



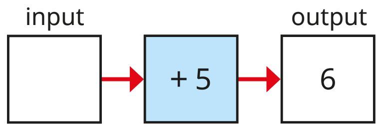
Input	5	23	5.1	23.2	0	−3	−5
Output							

- Complete the table for the function machine.



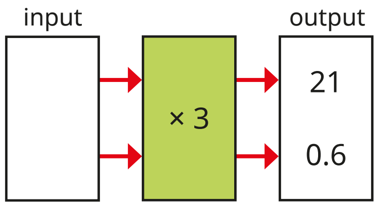
Input	3	10	0	2.5	0.25	7	70
Output							

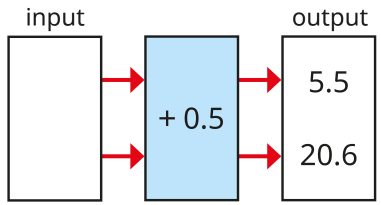
- The function machine shows the output, but not the input.



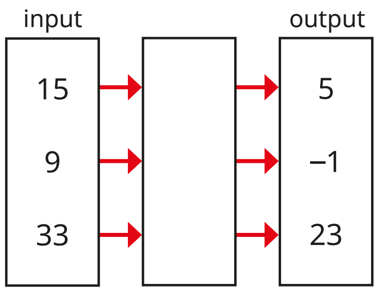
Talk to a partner about how you can work out the input.

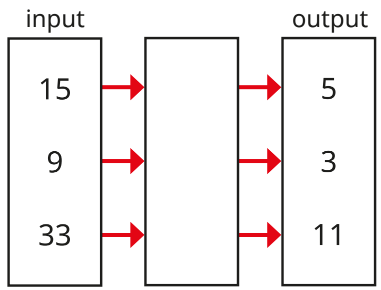
- Work out the missing inputs.





- What are the missing functions?





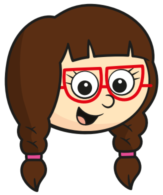
What do you notice?

1-step function machines

Reasoning and problem solving

Jo and Ron are working out the rule for the function machine.

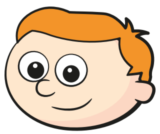
input		output
4 →		→ 44



The rule is + 40

Jo

The rule is × 11



Ron

Who do you agree with?

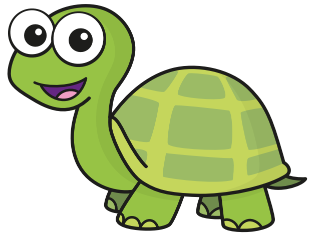
Explain your answer.

Either could be correct.

Tiny is working out the missing number.

Input	9	7	3.5	–2
Output	19	17	13.5	

The missing number is –12



Explain Tiny's mistake.

What is the missing number?

8

2-step function machines

Notes and guidance

In this small step, children move on to explore function machines with two steps.

As with 1-step machines, they start by looking at examples where the input is given and they need to find the output, using a mix of any of the four operations. Discuss why it is important that they follow the order of the functions; for example, the output of × 5 then + 3 will be different from + 3 then × 5

Children then move on to finding the input when the output is known by using the inverse of each function, recognising that they need to start with the second function when working backwards.

Children then look at problems where the input and output are given, but one of the two functions is missing. They may choose to do this problem working forwards or backwards.

Things to look out for

- Children may not follow the order of the functions, and it is important to explore the effect this can have.
- When finding the input, children may do the inverse of the first function first.

Key questions

- Which function should you apply first?
- What happens if you do not follow the functions in the correct order?
- What is the inverse of ______?
- When given the output, which function should you do first?
- What is the input if the output is ______?
- What is the missing function if the input is ______, the output is ____ and one of the functions is ______?
- Does it always matter what order you apply the functions?

Possible sentence stems

- First, I am going to ______, then I am going to ______
- If the input is ______, then the output is ______
- The inverse of ______ then ______ is ______ then ______

National Curriculum links

- Use simple formulae
- Find pairs of numbers that satisfy an equation with two unknowns
- Enumerate possibilities of combinations of two variables

2-step function machines

Key learning

- Here is a 2-step function machine.

  input → × 2 → + 1 → output

  - If the input is 5, what is the output?
  - If the input is 10, what is the output?

- Complete the tables for the function machines.

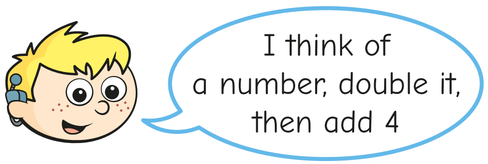
  input → + 2 → × 3 → output

Input	3	4	5	10
Output				

  input → × 3 → + 2 → output

Input	3	4	5	10
Output				

  What do you notice?

- 

  - What answer will Max get if he thinks of 20?
  - What number would Max need to think of to get the answer 20?

- Work out the missing inputs.

  input → + 10 → − 3 → output

input	output
	8
	23

  input → × 4 → + 5 → output

input	output
	13
	45

- What are the missing functions?

  input → × 5 → ? → output

input	output
3	16
25	126

  input → ? → ÷ 2 → output

input	output
29	10
11	1

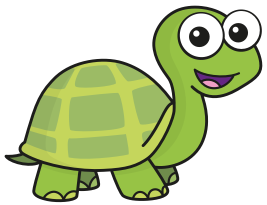
2-step function machines

Reasoning and problem solving

Tiny is using a 2-step function machine.

input → × 4 → + 2 → output

You can multiply numbers in any order, and you can add numbers in any order. This means you can solve this function machine in any order.

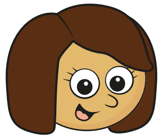


Do you agree with Tiny?

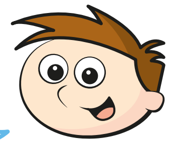
Explain your answer.

No

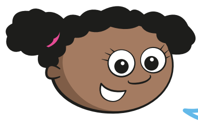
input			output
2			4
10			20



Kim: The missing functions are × 4 and ÷ 2



Teddy: The missing functions are × 2 and × 1



Whitney: There only needs to be one function, which is × 2

Who do you agree with?

What other functions would work?

They are all correct.

Form expressions

Notes and guidance

This small step is children's first experience of forming algebraic expressions using letters to represent numbers.

Children learn that phrases such as "2 more than a number" can be written more simply as, for example, "$x + 2$" or "$y + 2$". They also learn the convention that, for example, "$3t$" means 3 multiplied by t; as multiplication can represent repeated addition, this is also a simpler way of writing $t + t + t$. They use cubes and base 10 ones to represent expressions, with each cube representing an unknown number, x (or any letter), and the ones representing known numbers.

Children then revisit function machines, where x (or any letter) can represent the input. Discuss why it is not important at this stage to know what x represents, and that it could be any number input into the function machine.

Bar models can also be used to support children's understanding.

Things to look out for

- Children may assume that certain letters always represent specific numbers, for example a means 1, b means 2, c means 3 and so on.
- Children may not see $a \times 3$ and $3a$ as the same thing.

Key questions

- What could x represent?
- How can you represent this expression using a bar model?
- How else can you write $a + a$?
- What is the same and what is different about the expressions $x + 5$ and $5x$?
- If the input is p, what is the output?
- If m is the input, what is the output after the first operation? What is the output after the second operation?

Possible sentence stems

- ______ more than x can be written as ______
- ______ + ______ + ______ = 3 × ______ = ______
- If I have ______ x and I add/subtract ______ x, then I have ______ x altogether.

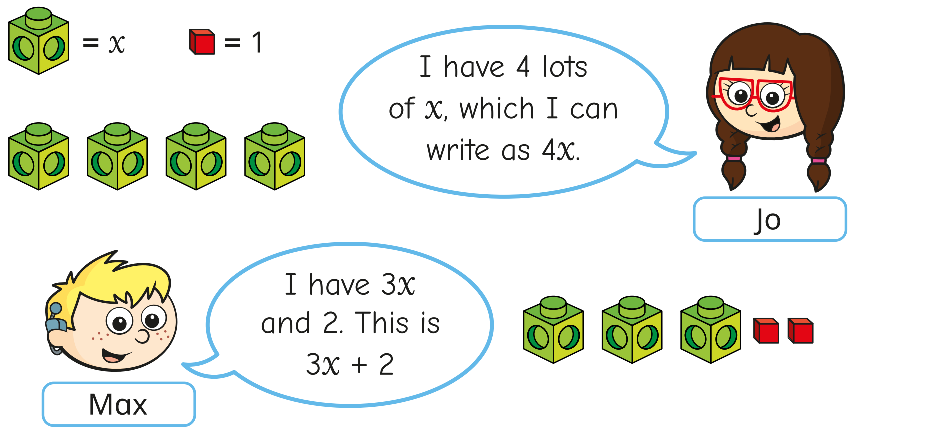
National Curriculum links

- Use simple formulae
- Express missing number problems algebraically

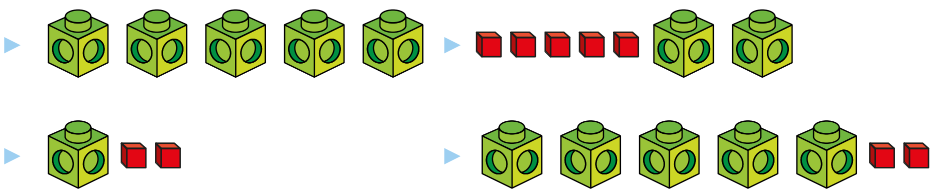
Form expressions

Key learning

- Jo and Max are using cubes to represent unknown numbers and base 10 ones to represent 1



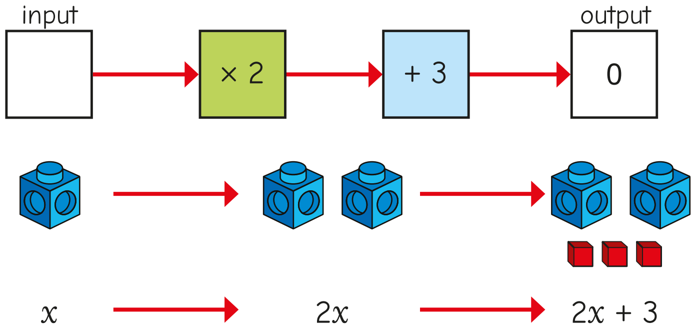
Use Jo and Max's method to write algebraic expressions for each set of cubes and base 10 ones.



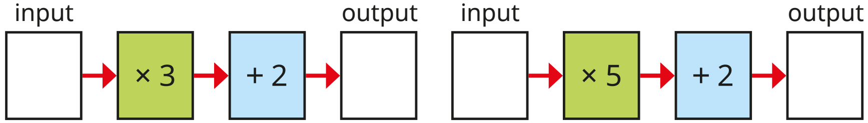
- Use cubes and base 10 to represent the algebraic expressions.

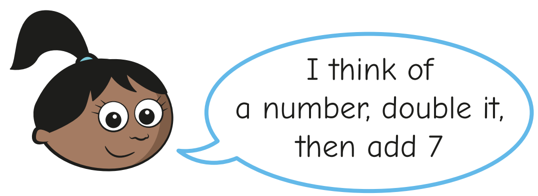
$y + 3$	$2y + 1$	$5 + 5y$

- Dan writes an expression for the 2-step function machine.



Use Dan's method to write an expression for each function machine.



- 

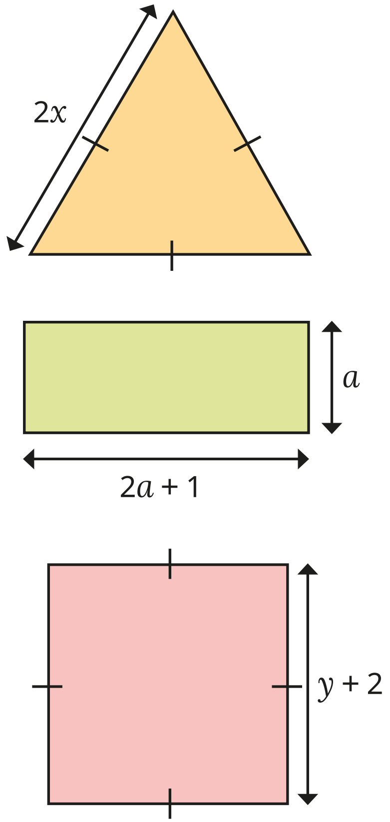
Sam calls the number she first thinks of x.

Write an expression for the number that Sam is thinking of after she has done the two calculations.

Form expressions

Reasoning and problem solving

Write expressions for the perimeters of the shapes.



$6x$

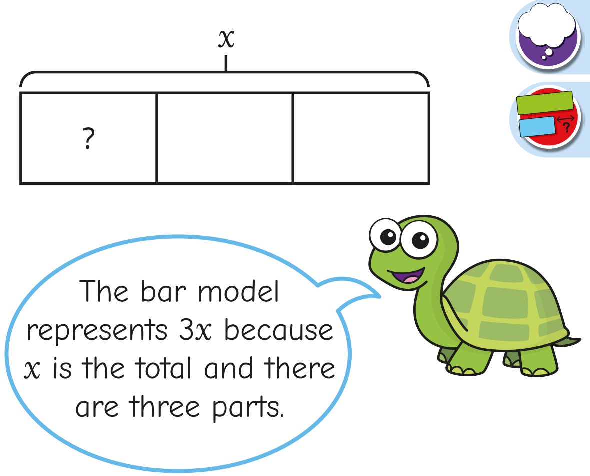
$6a + 2$

$4y + 8$

The perimeter of a rectangle is $12x$.

What could the sides of the rectangle be?

multiple possible answers, e.g.
$5x$ and x



Do you agree with Tiny?

Explain your answer.

No

Each part is $x \div 3$

Substitution

Notes and guidance

In this small step, children find values of expressions by substituting numbers in place of the letters.

Children should understand that the same expression can have different values depending on what number is substituted into it. Before working with letters, children explore concrete and pictorial representations. By assigning values to, for example, a square and a triangle, they can work out square + triangle. Similarly, building on representations from the previous step, if they assign a value to a cube, they can work out the value of an expression.

Children then move on to substituting numbers into abstract algebraic expressions such as $3a + 1$. This can be linked to the earlier learning of function machines, and thought of as "multiply by 3 and then add 1", or bar models, replacing each occurrence of the letter with its value.

Things to look out for

- Children may think that a is always equal to 1, b always equal to 2 and so on.
- If $a = 3$, children may see $2a$ as 23 rather than $2 \times 3 = 6$
- Children may misinterpret expressions such as $2a + 3$ as $5a$.

Key questions

- If 1 cube is worth ______, what are 3 cubes worth?
- What does $4x$ mean? If you know the value of x, how can you work out the value of $4x$?
- What does "substitute" mean?
- How can you represent the expression as a bar model? Which parts of the bar model can you replace with a number? What is the total value of the bar model?
- Which part of the expression can you work out first? What is the total value of the expression?

Possible sentence stems

- If ______ is worth ______, then ______ is worth ______
- To work out the value of ______, I need to replace the letter ______ with the number ______ and then calculate ______

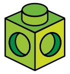
National Curriculum links

- Use simple formulae
- Express missing number problems algebraically

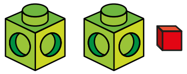
Substitution

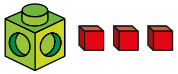
Key learning

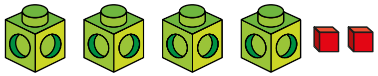
- Ann gives values to these cubes.

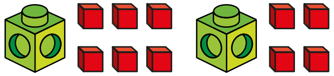
   = 5  = 1

  Work out the values of the sets of cubes.

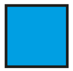
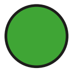
  ▶ 

  ▶ 

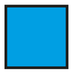
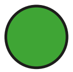
  ▶ 

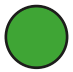
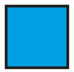
  ▶ 

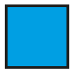
- Tom draws three shapes and gives each one a value.

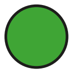
   = 3  = 4  = 5

  Work out the values of the expressions.

  ▶  + 

  ▶  – 

  ▶ 2 × 

  ▶  × 

- Here are three expressions.

$5a$	$a + 5$	$10 - a$

  ▶ Which expression has the greatest value when $a = 1$?

  ▶ Which expression has the greatest value when $a = 5$?

  ▶ Which expression has the greatest value when $a = 10$?

- Esther generates a sequence by substituting $n = 1$, $n = 2$, $n = 3$, $n = 4$ and $n = 5$ into the expression $4n + 1$

  When $n = 1$,

  $4n + 1 = 4 \times 1 + 1 = 4 + 1 = 5$

  Work out the other numbers in Esther's sequence.

  What patterns can you see?

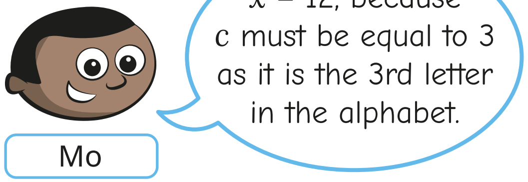
- If $a = 5$ and $b = 12$, work out the values of the expressions.

$a + b$	$b - a$	$2b - a$	$a + 3b$	$b \div 2$	$20 \div a$

Substitution

Reasoning and problem solving

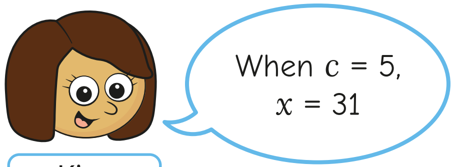
$x = 2c + 6$



Mo

Is Mo correct?

Explain why.



Kim

Explain why Kim is wrong.

What is the correct value of x when c = 5?

No

Kim has put the 2 next to the 5 to make 25, instead of multiplying 2 by 5

x = 16

Work out the missing values in the table.

x	$3x$	$3x + 5$
1		
3		
	12	
	36	
		20
		26

row 1: 3, 8

row 2: 9, 14

row 3: 4, 17

row 4: 12, 41

row 5: 5, 15

row 6: 7, 21

Find the value of c when a = 10

$p = 2a + 5$

$c = 10 - p$

$c = -15$

Formulae

Notes and guidance

In this small step, children are introduced to formulae using symbols for the first time, although they will be familiar with the idea of a formula in words, for example area of a rectangle = length × width.

Building on the previous steps, children substitute into formulae to work out values, noticing the effect that changing the input has on the output. Looking at familiar relationships between two or more variables will help to develop children's understanding, for example the number of days in a given number of weeks, the number of legs on a given number of insects and so on.

Children should recognise the difference between a formula and an expression, noticing that an expression does not have the equals sign, but a formula does.

Things to look out for

- Children may mix up the variables in a formula, for example using $w = 7d$ to represent the formula for the number of days in a given number of weeks.

Key questions

- What is a formula?
- What formulae do you know?
- How is a formula similar to/different from an expression?
- What is the formula for ______?
- If the formula is $t = 3s + 1$ and you know that $s =$ ______, how can you work out t?
- Which letter(s) do you know the value of? Which letter(s) can you work out?

Possible sentence stems

- In the formula ______, the letter ______ represents ______ and the letter ______ represents ______
- To work out ______ when I know ______, I substitute ______ into the formula.

National Curriculum links

- Use simple formulae
- Express missing number problems algebraically

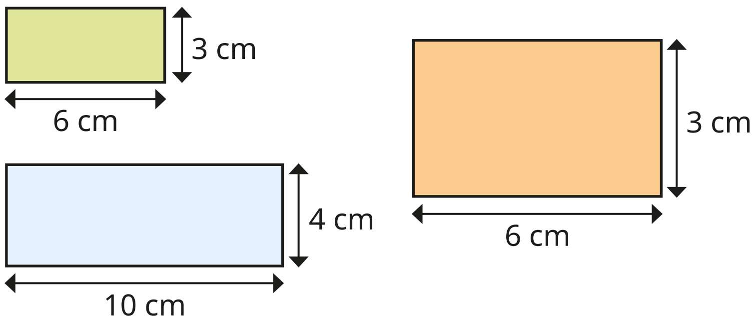
Formulae

Key learning

- Ron uses a formula to work out the areas of rectangles.

  $A = lw$
  When $l = 7$ and $w = 4$, $A = 7 \times 4 = 28$

  - What do the letters A, l and w represent?
  - Use the formula to find the areas of the rectangles.

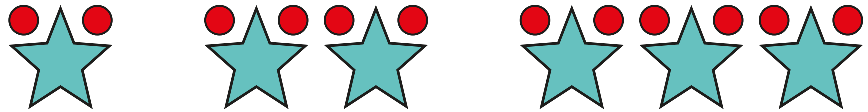


- The time taken to cook a turkey is 90 minutes, plus an additional 20 minutes for every kilogram of turkey.

  This can be written as the formula $T = 90 + 20m$

  - What do the letters T and m represent?
  - Use the formula to work out the time to cook:
    - a 3 kg turkey
    - a 10 kg turkey

- Fay makes a sequence of patterns with stars and circles.



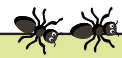
Complete the table to show the number of circles and stars in the patterns.

Number of stars	1	2	3	5		
Number of circles	2				18	30

If s = number of stars and c = number of circles, which formula describes Fay's pattern?

$s = 2 + c$	$c = s + 2$	$c = 2s$	$s = 2c$	$2s = c + 2$

- The table shows the total number of legs on a given number of ants.

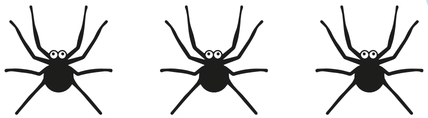


Number of ants (a)	1	2	3		
Number of legs (L)	6			30	72

Complete the table and write a formula that describes the pattern.

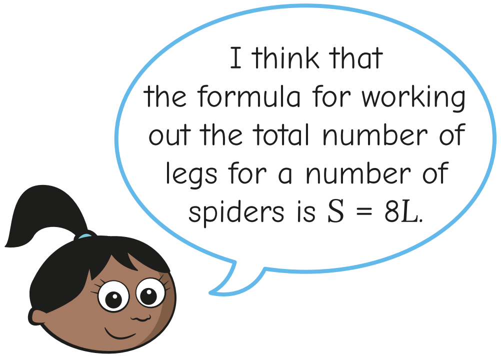
Formulae

Reasoning and problem solving



S = number of spiders

L = total number of legs



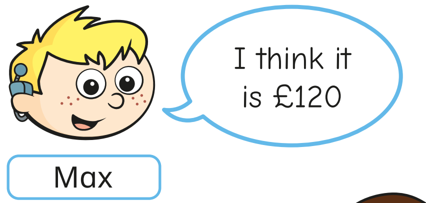
Do you agree with Sam?

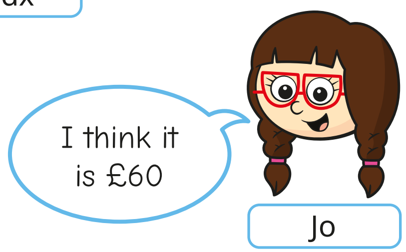
Explain your answer.

No

Max and Jo use this formula to work out the cost in pounds (C) of four hours (h) of cleaning.

$$C = 20 + 10h$$





Who do you agree with?

Explain your answer.

Jo

Form equations

Notes and guidance

In this small step, children form equations from diagrams and word descriptions.

Begin the step by looking at the difference between an algebraic expression and an equation. An expression, such as $2x + 6$, changes value depending on the value of x, whereas in an equation, such as $2x + 6 = 14$, x has a specific value. You may need to remind children of the algebraic conventions learnt earlier in the block, for example writing $a + a + a$ (or $a \times 3$) as $3a$ and "4 more than b" as $b + 4$

Various representations can be used to support children's understanding, including bar models, part-whole models and cubes and counters with a designated value. It is important that children understand that, for example, the letter c represents the numerical value of the cube rather than the cube itself.

Things to look out for

- Children may look to work out the value rather than represent the information as an equation.
- Children may make errors using algebraic notation, for example confusing $3x$ and $x + 3$

Key questions

- If a is a number, how do you write "3 times the value of a"?
- How do you write "4 more than the number x"?
- If 4 more than the number x is equal to 26, how can you write this as an equation?
- Is an equation the same as or different from a formula?
- What is the difference between an equation and an expression?
- Can you write the equation a different way?
- Is ______ an equation or an expression? How do you know?

Possible sentence stems

- ______ + ______ +______ = 3 × ______ = ______
- The equation ______ means that the expression ______ is equal to ______
- ______ more/less than ______ is equal to ______ can be written as the equation ______ = ______

National Curriculum links

- Express missing number problems algebraically

Form equations

Key learning

- Tom thinks of a number and calls it x.

  Which expression represents 5 more than Tom's number?

$5x$	$x + 5$	$x - 5$	$x \div 5$

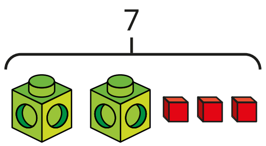
  Double Tom's number is 64

  Which equation shows this information?

$x + 2 = 64$	$x \div 2 = 64$	$2x = 64$	$x - 2 = 64$

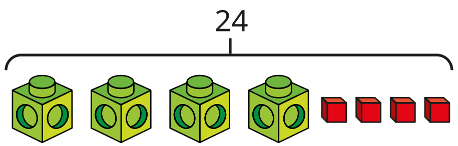
- Max has represented some equations.

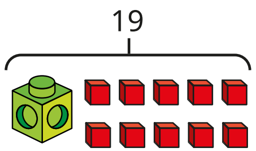
  Each linking cube represents y and each base 10 cube represents 1



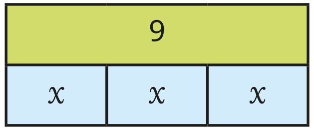
$2y + 3 = 7$

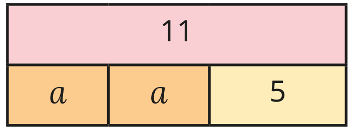
  What equations are represented?

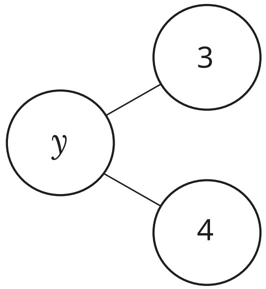


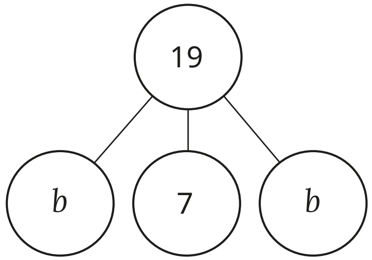


- Write equations to match the models.









- A book costs £5 and a magazine costs £n.

  The total cost of the book and the magazine is £8

  Write this information as an equation.

- Write algebraic equations for the word problems.
  - I think of a number and subtract 17. My answer is 20
  - I think of a number and multiply it by 5. My answer is 45

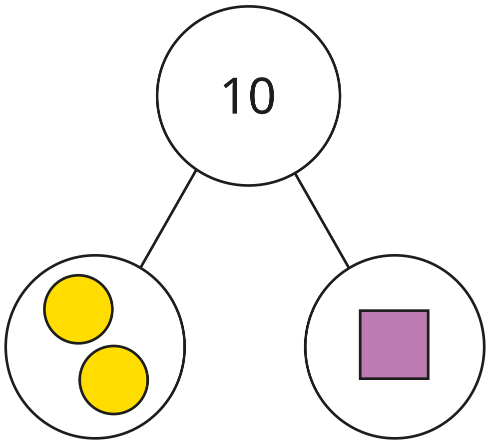
- Draw bar models to represent the equations.

$x + 5 = 11$	$2y = 15$	$3a + 9 = 30$

Form equations

Reasoning and problem solving

Here is a part-whole model.



Write an equation representing the part-whole model.

Each shape has a different integer value.

What values might the shapes have?

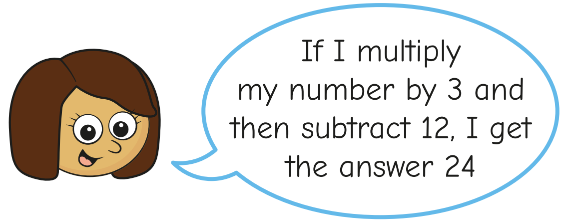
Using c for circles and s for squares:

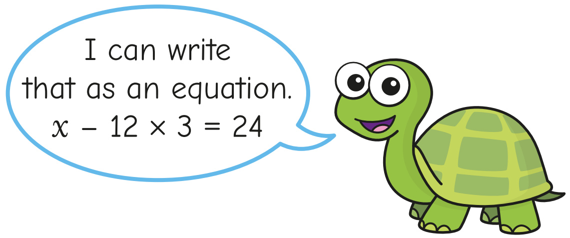
$2c + s = 10$

multiple possible answers, e.g.

$c = 2, s = 6$

Kim is thinking of a number.





What mistake has Tiny made?

Write the correct equation for Kim's problem.

Tiny has not applied the operations in the correct order.

$3x - 12 = 24$

Solve 1-step equations

Notes and guidance

In this small step, children look at solving equations formally for the first time. At first, they might find the notation a bit confusing, but encourage them to consider equations as a different way of writing "missing number" problems.
For example, $x + 5 = 12$ is the same as ______ + 5 = 12

It is useful to begin by looking at "think of a number" questions, such as "Mo thinks of a number, adds 7 and gets the answer 20. What was his original number?" and relating this to the equation $n + 7 = 20$. Similarly, you can build on earlier learning using function machines, relating finding an input for a given output to solving the corresponding equation. In both cases, children should see that using inverse operations helps to solve the equations.

Things to look out for

- Children may not use the inverse operation to solve an equation, for example $x + 3 = 5$, so $x = 8$
- Children may think that the values of letters are permanently fixed. For example, having solved an equation for x, they may apply this value for x to a different equation.

Key questions

- What does the expression $3x$ mean?
- If you know 3 times the value of a number, how can you use this to work out the number?
- How can you represent the problem as a bar model?
- How can you represent the problem as an equation?
- What is the inverse of ______?
- What does the bar model show?
  What can you use it to work out?
- How can you draw a function machine to represent the equation?
  How does the function machine help you to solve the equation?

Possible sentence stems

- The inverse of ______ is ______
- If ______ has been added to a number to give ______, then to work out the number I need to ______ from ______

National Curriculum links

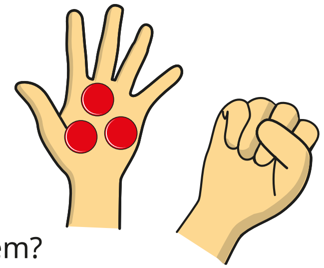
- Express missing number problems algebraically

Solve 1-step equations

Key learning

- Ben has 9 counters altogether.

  He has 3 counters in his left hand and c counters in his closed right hand.



  Which equation represents this problem?

$c - 3 = 9$	$3c = 9$	$c + 3 = 9$	$c = 9 + 3$

  How many counters does he have in his closed hand?

- Fay thinks of a number.

  She adds 9 to her number.

  She gets the answer 15

  What was her original number?

  Explain how the equation $x + 9 = 15$ represents this problem.

- Dan thinks of a number and multiplies it by 3 to get the answer 12

  Which equation shows this?

$3x = 12$	$3 + x = 12$	$x - 3 = 12$	$x \div 3 = 12$

  What was Dan's original number?

- Write expressions for the outputs of the function machines.

input		output
x	$+ 9$	

input		output
y	$- 3$	

input		output
p	$\times 5$	

input		output
t	$\div 2$	

  If the output of all the machines is 20, write and solve equations to find the values of the letters.

- Write an equation to represent each bar model.

  Then find the value of x for each one.

15		
x	x	x

12	
x	7

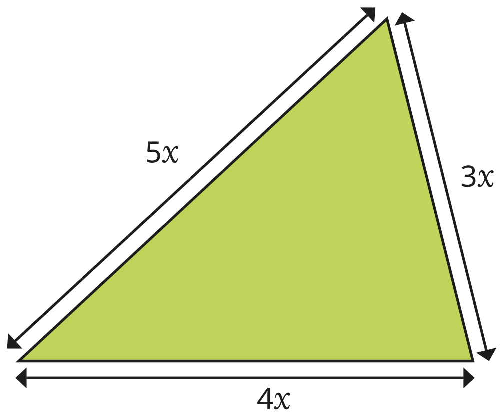
- Solve the equations.

$3x = 21$	$y + 5 = 11$	$z - 6 = 8$	$p \div 3 = 10$

Solve 1-step equations

Reasoning and problem solving

The perimeter of the triangle is 216 cm.



Form an equation to find the value of x.

Work out the lengths of the sides of the triangle.

$12x = 216$

$x = 18$

54 cm, 72 cm and 90 cm

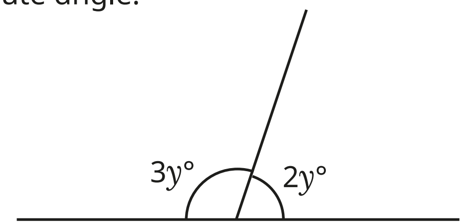
$x - 9 = 0$

Do you agree with Tiny?

Explain your answer.

No

Work out the size of the acute angle.



How can you check your answer?

72°

Solve 2-step equations

Notes and guidance

In this small step, children move on to solving equations with two steps.

As with 1-step equations, initially equations of this type can be represented by 2-step "think of a number" problems and/or function machines, where children work backwards using inverse operations to find the original number or input. They can then link this to finding an unknown in a 2-step equation.

Children can also use concrete resources to represent the problems and to work out missing numbers. Bar models are another useful representation, as they give a visual clue to the steps needed to work out the unknowns. It is useful to have the abstract representation alongside the models to help develop understanding.

Things to look out for

- Children may think the values of letters are permanently fixed. For example, having solved an equation for x, they may apply this value for x to a different equation.
- When "working backwards" to solve equations, children may do the inverse operations in the wrong order.

Key questions

- If you know 3 more than $2x$, how can you work out $2x$?
- If you know 5 less than $2x$, how can you work out $2x$?
- How can you represent the problem with a bar model? Which part(s) of the bar model do you already know? Which part(s) can you work out?
- How can you represent the problem with an equation? What is the first step you need to take to solve the equation?
- How can you represent the equation using a function machine? How can you use the function machine to help you solve the equation?

Possible sentence stems

- If ______ x + ______ = ______, then ______ x = ______, so x = ______
- The first step in solving the equation is to ______
  The second step in solving the equation is to ______

National Curriculum links

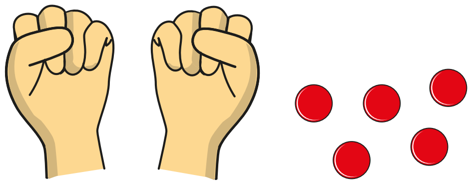
- Express missing number problems algebraically

Solve 2-step equations

Key learning

- Tommy has 17 counters.

  He puts the same number of counters (c) in each hand and has some left over.

  Which equation shows this?

  $c + 2 = 5$ | $2c = 17$ | $2c + 5 = 17$ | $2c + 17 = 5$

  Solve the equation to work out how many counters Tommy has in each hand.

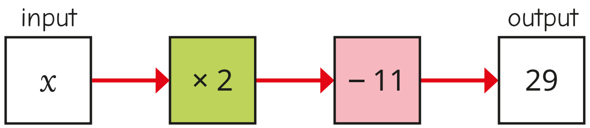
- Kay thinks of a number.

  She multiplies the number by 2 and then adds 5

  She gets the answer 29

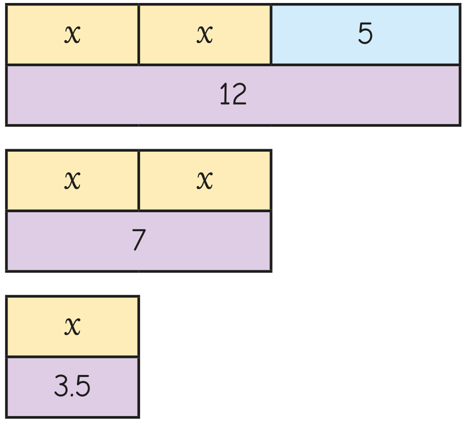
  What number did Kay think of?

- Explain how this 2-step function machine shows the equation $2x - 11 = 29$

  Work out the value of x.

- Ron uses a bar model to solve an equation.

  $2x + 5 = 12$

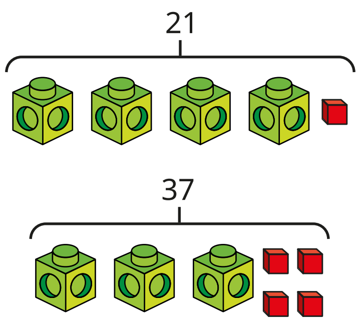
  $2x = 7$

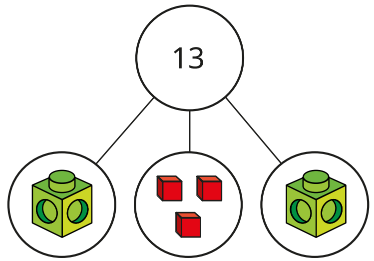
  $x = 3.5$

  Use Ron's method to solve the equations.

  $3b + 4 = 19$ | $20 = 4b + 2$

- Write and solve equations for the models.

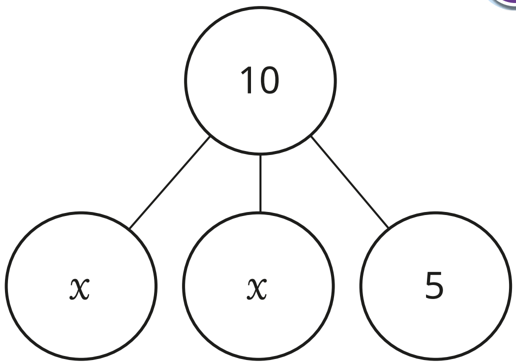
  

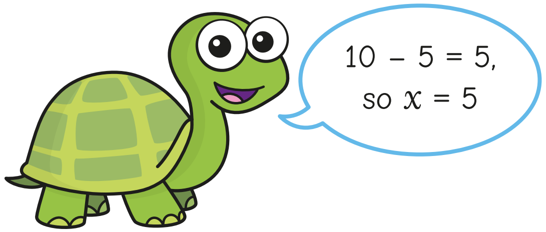
  

Solve 2-step equations

Reasoning and problem solving

Tiny is working out the value of x.



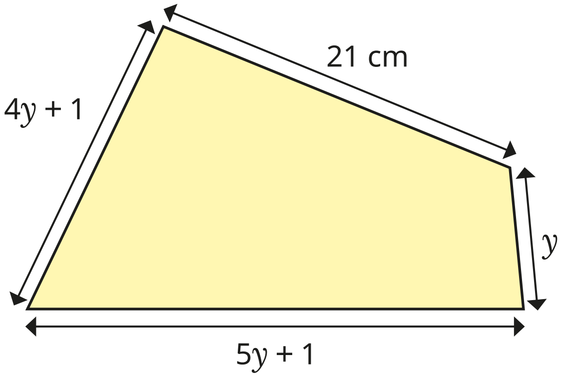


Do you agree with Tiny?

Explain your reasoning.

No

The perimeter of the quadrilateral is 83 cm.



Work out the value of y.

Explain your method to a partner.

$y = 6$ cm

Find pairs of values

Notes and guidance

In this small step, children explore equations with two unknown values, recognising that these can have several possible solutions.

Children can use substitution to work out pairs of possible values. For example, if $x + y = 9$, they find the values of y for different values of x. They should work systematically to find all the possible integer values. A table is a good way to support this. In this step, the possible values will always be integers greater than or equal to zero, but this could be extended to negative and decimal values. Begin with simple equations of the form $x + y =$ ______ or $ab =$ ______, before moving on to more complex equations that include multiples of the unknowns, for example $2x + 3y =$ ______

It is important that children understand that they cannot know the exact value of the two unkowns, as they do not have enough information.

Things to look out for

- Children may not consider zero as a possible value for one of the unknowns.
- Children may need support to work systematically to find all possible solutions.

Key questions

- What two numbers could add together to make ______?
- What could the values of x and y be in the equation ______?
- Why are there several possible answers for this question?
- Have you found all the possible pairs of values? How do you know?
- In the equation ______, if $x =$ ______, what must the value of y be? If x is a different value, does y also change?
- How can you draw a bar model to represent the equation ______?

Possible sentence stems

- In the equation $x + y =$ ______, if $x =$ ______ then $y =$ ______
- If the product of p and q is ______, then p could be ______ and q could be ______

National Curriculum links

- Find pairs of numbers that satisfy an equation with two unknowns
- Enumerate possibilities of combinations of two variables

Find pairs of values

Key learning

- x and y are both whole numbers.

  $x + y = 5$

  Ann creates a table to work out the possible sets of values of x and y.

x	y	$x + y$
0	5	5
		5
		5
		5
		5
		5

  Work systematically to complete Ann's table.

- a and b are both whole numbers.

  $a \times b = 24$

  Create a table to show all the possible sets of values for a and b.

- p and q are both whole numbers less than 12

  $p - q = 3$

  Find all the possible values of p and q.

- x and y are both whole numbers.

  $x > y$ $\quad$ $x + y = 25$

  - If x is odd and y is even, what are the possible pairs of values for x and y?
  - If x and y are both even, what are the possible pairs of values for x and y?
  - If x is a multiple of 5 and y is even, what are the possible pairs of values for x and y?

  Create your own problem like this for a partner.

- a and b are integers.

  $3a + 2b = 20$

  Work out three possible pairs of values for a and b.

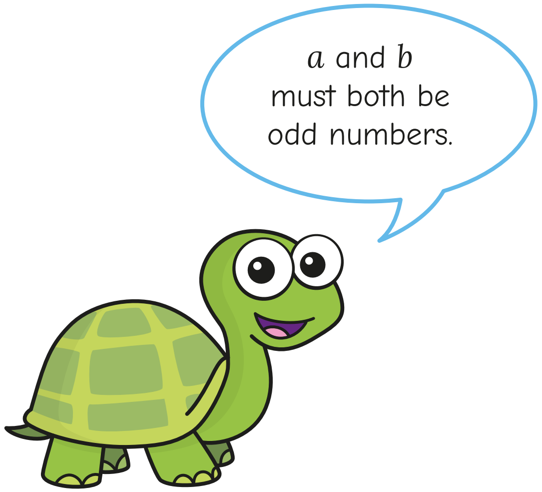
  Compare methods with a partner.

Find pairs of values

Reasoning and problem solving

a and b are both whole numbers.

$$ab + b = 18$$



Is Tiny correct?

Explain your answer.

No

a, b and c are integers between 0 and 5

$$a + b = 6 \qquad b + c = 4$$

Find the values of a, b and c.

How many possibilities can you find?

$a = 2$, $b = 4$, $c = 0$

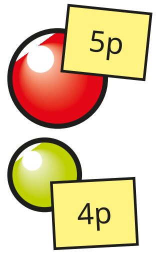
$a = 3$, $b = 3$, $c = 1$

$a = 4$, $b = 2$, $c = 2$

$a = 5$, $b = 1$, $c = 3$

Large beads costs 5p and small beads cost 4p.

Sam spends 79p on beads.



What possible combinations of large beads (l) and small beads (s) could she buy?

Write each possible combination as an expression.

$3l + 16s$

$7l + 11s$

$11l + 6s$

$15l + s$

Solve problems with two unknowns

Notes and guidance

Building on previous learning, in this small step children solve problems with two unknowns when more than one piece of information is given, so there is only one possible solution.

Examples include the case where the sum and the difference of both unknowns is given. Bar models are used throughout the step to represent problems and to support children's understanding.

Other structures are also explored, including where one of the unknowns is a multiple of the other. In this case, a bar model can be used to work out the values of the numbers if either their total or their difference is known. Finally, children look at equations with two unknowns where the coefficient of only one of the unknowns is different, for example $x + 2y = 17$ and $x + 5y = 38$. Again, a bar model will help children to see why $3y$ must be equal to 21, after which y and x can be found.

Things to look out for

- Children may use trial and error rather than a bar model approach.
- Children may think that there are several possible solutions, as in the last step.

Key questions

- How can you represent this information as a pair of equations?
- How can you represent this information with a bar model?
- What information does the bar model show? What else can you work out?
- How can you draw a bar model to represent the problem? Which parts can you label straight away? What else can you work out?
- Is there more than one possible solution?

Possible sentence stems

- If ______ lots of x is worth ______, then x = ______ ÷ ______ = ______
- If I know the value of ______, I can find the value of ______ by substituting into the equation ______

National Curriculum links

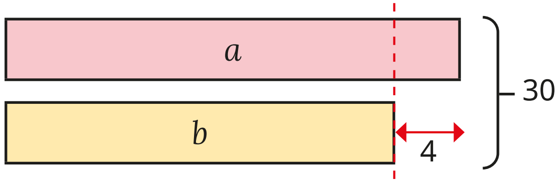
- Express missing number problems algebraically
- Find pairs of numbers that satisfy an equation with two unknowns

Solve problems with two unknowns

Key learning

- The sum of a and b is 30

  The difference between a and b is 4



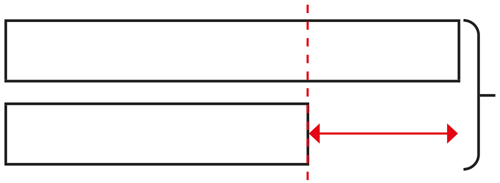
  Use the bar model to work out the values of a and b.

- Here is some information about two numbers, x and y.

  $x + y = 10$

  $x - y = 2$

  - Label the information on the bar model.



  - Use the bar model to work out the values of x and y.

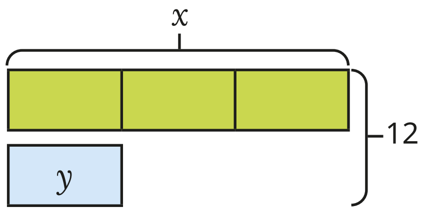
- The sum of two numbers, p and q, is 55

  The difference between p and q is 7

  Show this as a bar model and find the values of p and q.

- The sum of x and y is 12

  x is 3 times the size of y.



  - Explain how you can use the bar model to work out the value of y.
  - What is the value of x?

    Are there any other possible solutions?

- The sum of two numbers, a and b, is 18

  a is one-fifth the size of b.

  Draw a bar model to represent this problem and work out the values of a and b.

- Tom and Ann both go for a walk.

  Between them they walk 21 km.

  Tom walks 6 times as far as Ann does.

  How much further does Tom walk than Ann?

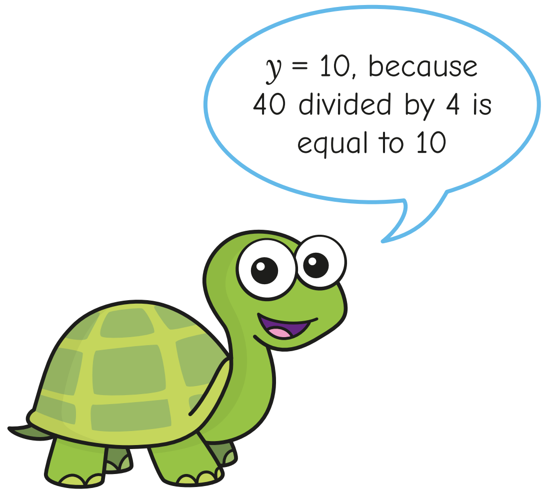
Solve problems with two unknowns

Reasoning and problem solving

The sum of x and y is 40

x is 4 times the size of y.

What is the value of y?



Show that Tiny is wrong.

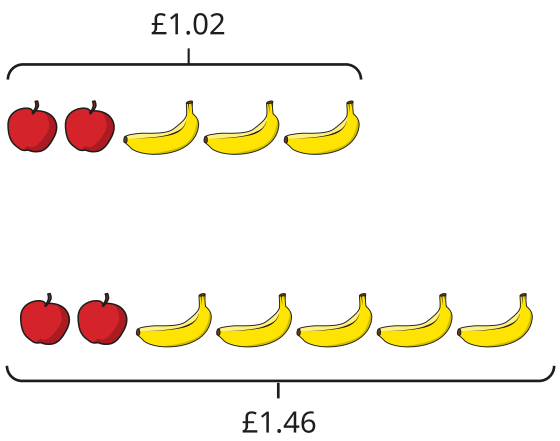
Find the correct values of x and y.

If y = 10, x = 40 and $x + y$ = 50

y = 8 and x = 32

Two apples and three bananas cost £1.02

Two apples and five bananas cost £1.46



What is the total cost of one apple and one banana?

40p