Spring Block 1

Ratio

Small steps

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Step 2	Use ratio language
Step 3	Introduction to the ratio symbol
Step 4	Ratio and fractions
Step 5	Scale drawing
Step 6	Use scale factors
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Small steps

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Add or multiply?

Notes and guidance

In this small step, children explore the fact that the relationship between two numbers can be expressed additively or multiplicatively. For example, the relationship between 3 and 9 can be expressed as an addition (3 + 6 = 9) or a multiplication (3 × 3 = 9). Children use this understanding to complete sequences of numbers, deciding whether each relationship is additive or multiplicative.

Children also explore the inverse relationships related to each of these, for example 9 – 6 = 3 and 9 ÷ 3 = 3. Using language such as "3 times the size" and "a third of the size" will support their understanding of multiplicative relationships.

Children will explore these relationships using double number lines and should be encouraged to explore all of the additive and multiplicative links that can be seen.

Things to look out for

- Children may see just additive relationships and not notice the multiplicative relationships.
- Children may not start double number lines from zero.
- When using double number lines, children may focus on the horizontal relationships and not notice the vertical relationships.

Key questions

- How can you describe the relationship between these two numbers using addition/multiplication?
- What is the inverse of addition/multiplication?
- What addition/subtraction/multiplication/division calculations can be written from this information?
- Is the relationship in the sequence additive or multiplicative?
- How do the relationships on the upper number line relate to those on the lower number line?

Possible sentence stems

- _____ × _____ = _____ and _____ +_____ = _____
- _____ is _____ times the size of _____
- _____ is $\frac{\square}{\square}$ the size of _____

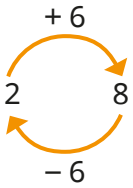
National Curriculum links

- Solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts

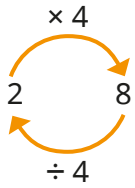
Add or multiply?

Key learning

- The relationship between 2 and 8 can be described as additive or multiplicative.

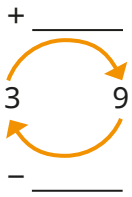


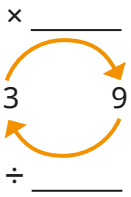
8 is 6 more than 2
2 is 6 less than 8

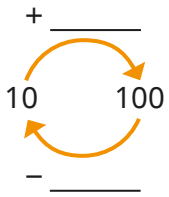


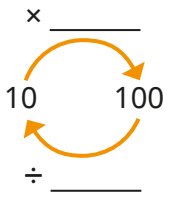
8 is four times the size of 2
2 is a quarter the size of 8

Complete the models to show the additive and multiplicative relationships.









Describe the relationships to a partner.

- A sequence starts 3, 6 ...
  - Explain why the next number could be 9
  - Explain why the next number could be 12
  - What could the next number be in these sequences?

5, 10 ...	7, 21 ...	100, 50 ...

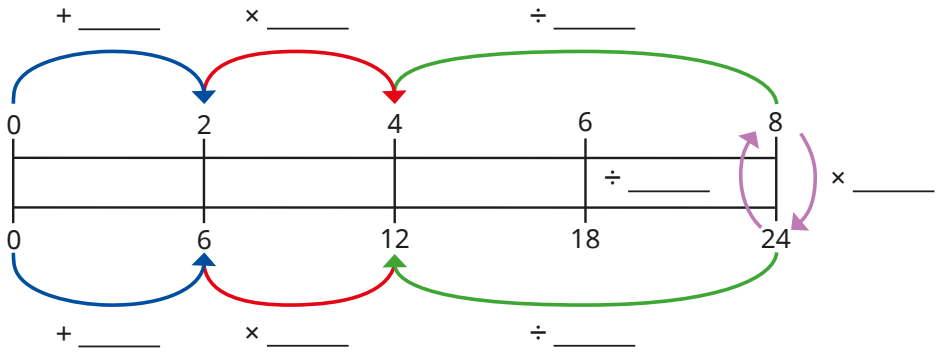
Find two answers for each.

- Complete the sequences.
  - 4, 8, _____, 32, _____, _____
  - _____, 14, 21, 28, _____, _____
  - 1, _____, _____, 27, 81, _____

Are the relationships additive or multiplicative? Could they be both?

- The double number line shows the relationship between two sets of numbers.

Fill in the missing values to describe the relationships.

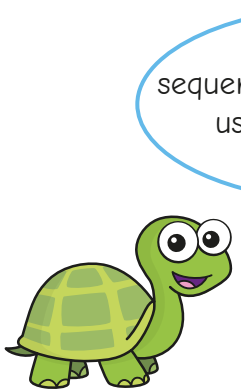


What other additive and multiplicative relationships can you see on the double number line?

Add or multiply?

Reasoning and problem solving

6	12		
2	4	8	
4	12	20	



Do you agree with Tiny?

Explain your answer.

No

Here are the different options in a pizza shop.

Base	Topping
Thin	Cheese and tomato
Deep pan	Vegetarian feast
	Chicken
	Meat feast

Use both additive and multiplicative reasoning to explain why there are 8 possible combinations of base and topping.

The restaurant introduces a new topping of tuna and sweetcorn.

How many combinations are there now?

How many combinations would there be with 4 base options and 17 topping options?

Did you use additive or multiplicative relationships to work out each answer?

10 | 68

Use ratio language

Notes and guidance

In this small step, children are introduced to the idea of ratio representing a multiplicative relationship between two amounts.

Children see how one value is related to another by making simple comparisons, such as: "For every 2 blue counters, there are 3 red counters." A double number line can be used to show such relationships, building up to recognise that this example is equivalent to 4 blue, 6 red or 20 blue, 30 red and so on. At this point, relationships will only be expressed in words and the ratio symbol will be introduced in the next step.

Children move on to expressing relationships more simply. For example, if there are 10 red and 15 blue counters, these can be physically rearranged so that "For every 2 red counters, there are 3 blue counters." Children can link this to dividing by a common factor, 5, and relate this to their understanding of simplifying fractions.

Things to look out for

- Children may use additive rather than multiplicative relationships to make comparisons, for example "There is one more blue than red."

Key questions

- How can you give the relationship between the number of _____ and the number of _____?
- For every _____, how many _____ are there?
- How can you rearrange the counters to make the ratio simpler?
- What number is a common factor of _____ and _____? How can you use this to make the ratio simpler?
- How many _____ would there be if there were _____?

Possible sentence stems

- For every _____, there are _____
- If there were _____, there would be _____
- A common factor of _____ and _____ is _____

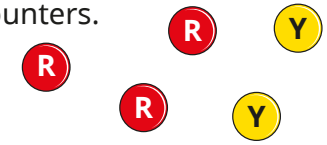
National Curriculum links

- Solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts

Use ratio language

Key learning

- Complete the sentences to describe the counters.

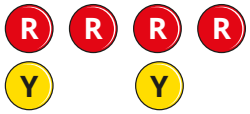


There are _____ red counters and _____ yellow counters.

For every _____ red counters, there are _____ yellow counters.

For every _____ yellow counters, there are _____ red counters.

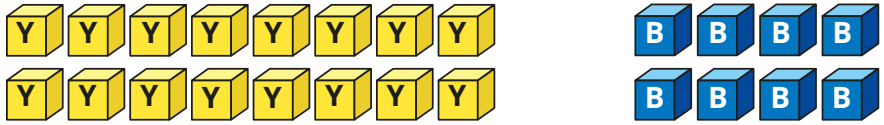
- Complete the sentence to describe the counters.



For every _____ red counters, there is _____ yellow counter.

Can you complete it a different way?

- Complete the sentences to describe the cubes.

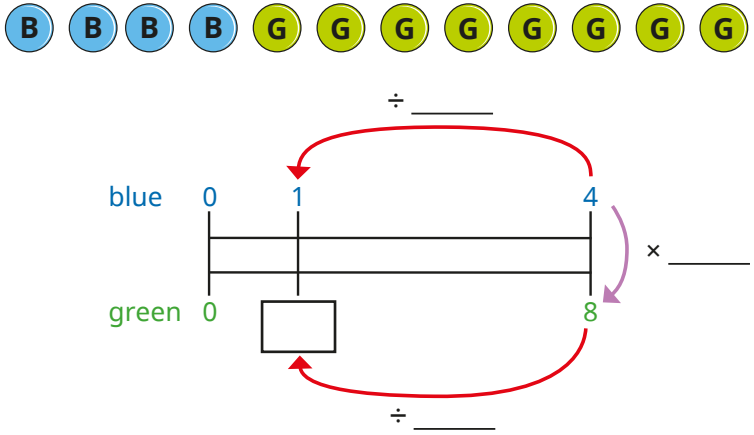


For every 16 yellow cubes, there are _____ blue cubes.

For every 8 yellow cubes, there are _____ blue cubes.

For every 1 blue cube, there are _____ yellow cubes.

- Amir is using a double number line to find equivalent ratios.



  - Use Amir's number line to help you complete the sentence.

    For every 1 blue counter, there are _____ green counters.

  - Use a double number line to complete the sentences.

    For every 4 green counters, there are _____ blue counters.

    For every _____ blue counters, there are 16 green counters.

- Complete the sentences to describe the fruit.

For every _____ pears, there are _____ bananas.

For every _____ pears, there are _____ apples.

Use ratio language

Reasoning and problem solving

Jack puts red and yellow tiles in this pattern.

R	R	Y	Y	Y	R	R	Y	Y	Y

I have 16 more red tiles and 20 more yellow tiles.

Can Jack continue this pattern without there being any tiles left over?

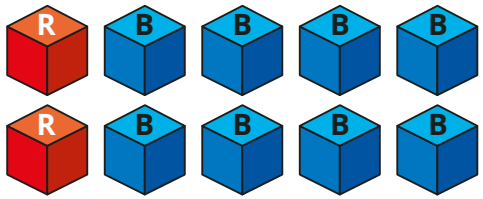
Explain your answer.

No

There are 2 red tiles for every 3 yellow tiles.

16 red tiles will need 24 yellow tiles.

Decide if each statement is true or false.



For every red cube, there are 8 blue cubes.

For every 4 blue cubes, there is 1 red cube.

For every 3 red cubes, there would be 12 blue cubes.

For every 16 cubes, 4 would be red and 12 would be blue.

Give reasons for your answers.

False

True

True

False

Introduction to the ratio symbol

Notes and guidance

In this small step, children continue to explore the multiplicative relationship between values, now seeing it written using the ratio symbol, a colon.

Explain that the wording, "For every _____, there are _____" can be written as _____ : _____. Show children that the order in which the notation is used is important. For example, for every 2 red cubes there are 3 blue cubes, so red to blue is 2 : 3. For every 3 blue cubes, there are 2 red cubes, so blue to red is 3 : 2. Ensure that children know, and convey in their answers, which number refers to which value.

Children build on the ideas of the previous step to understand that the same ratio can be written in different forms, for example 4 : 6 can be written as 2 : 3. This step is a good opportunity to use contexts such as measure, looking at the ratios of the masses of ingredients in recipes.

Things to look out for

- Children may not understand the meaning of the ratio symbol, and may confuse it with a decimal point.
- When simplifying a ratio, children may try to use additive rather than multiplicative relationships.

Key questions

- If there are 3 blue counters and 5 red counters, how can you describe the relationship between these numbers?
- What does the : symbol mean in the context of ratio?
- What does 2 : 3 mean?
- How can you compare the relationship between three quantities?
- Are the ratios 2 : 3 and 3 : 2 the same?
- How else can you write the ratio 2 : 4?

Possible sentence stems

- For every _____, there are _____, which can be written as _____ : _____
- The ratio of _____ to _____ is _____ : _____
- In the ratio _____ : _____, the first number represents _____ and the second number represents _____

National Curriculum links

- Solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts

Introduction to the ratio symbol

Key learning

- Complete the sentences.

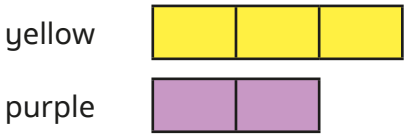
For every _____ red counters, there are _____ blue counters.

The ratio of red counters to blue counters is _____ : _____

For every _____ blue counters, there are _____ red counters.

The ratio of blue counters to red counters is _____ : _____

- Aisha draws a bar model to show the ratio of yellow to purple gummy bears.



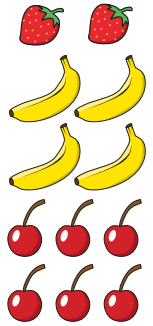
Complete the sentences.

The ratio of yellow gummy bears to purple gummy bears is _____ : _____

The ratio of purple gummy bears to yellow gummy bears is _____ : _____

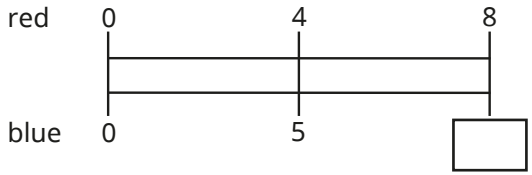
- Write the ratio of:
  - bananas to strawberries
  - cherries to strawberries
  - strawberries to bananas to cherries
  - cherries to strawberries to bananas

Draw a bar model to represent each ratio.



- Here are 8 red counters.

How many blue counters does he need so that the ratio of red to blue is 4 : 5?



How does the double number line help to work it out?

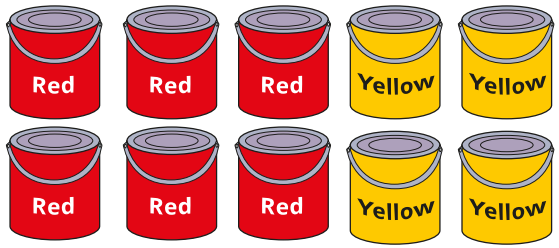
- Max has blue and red counters in the ratio 3 : 5

He has 15 blue counters.

How many red counters does he have?

Introduction to the ratio symbol

Reasoning and problem solving



Decide if each statement is true or false.

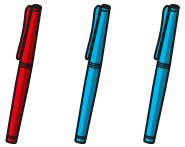
There are 2 yellow tins for every 3 red tins.	True
There are 2 red tins for every 3 yellow tins.	False
The ratio of red tins to yellow tins is 2 : 3	False
The ratio of yellow tins to red tins is 2 : 3	True

Explain your answers.

In a box, there are some red, blue and green pens.

The ratio of red pens to green pens is 3 : 5

For every 1 red pen, there are 2 blue pens.



There are 6 red pens in the box.

How many green pens are there?

How many blue pens are there?

Write the ratio of red pens to blue pens to green pens.

10

12

3 : 6 : 5

6 : 12 : 10

3 : 6 : 5

Ratio and fractions

Notes and guidance

In this small step, children explore the differences and similarities between ratios and fractions.

Children may have already noticed that simplifying ratios is similar to simplifying fractions and that both involve dividing by common factors. A possible misconception is thinking, for example, that the ratio 1 : 2 is the same as $\frac{1}{2}$. Exploring links between ratios and fractions using representations such as counters and bar models can help to overcome this. The key point is that a ratio compares one item with another, whereas fractions compare each part with the whole.

Children then explore ratio when given a fraction as a starting point. For example, they are told that $\frac{1}{4}$ of a group of objects is blue, and they need to find the ratio of blue to not blue. Initially, they may think the ratio is 1 : 4, but concrete resources and diagrams can support them to see it is 1 : 3

Things to look out for

- Children may not consider the whole when linking ratios and fractions. For example, they may think the 2 in 2 : 3 is $\frac{2}{3}$ rather than $\frac{2}{5}$

Key questions

- What is the ratio of one part to another?
- How many parts are there altogether?
- What fraction of the whole is the first/second/third part?
- How are fractions and ratios similar? How are they different?
- What fraction does the ratio 1 : 2 mean? Is this the same as $\frac{1}{2}$ or is it different?
- How can you represent the ratio/fraction as a bar model?

Possible sentence stems

- The ratio of _____ to _____ is _____ : _____

  There are _____ parts altogether.

  The fraction that is _____ is _____

National Curriculum links

- Solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts
- Solve problems involving unequal sharing and grouping using knowledge of fractions and multiples

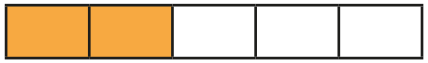
Ratio and fractions

Key learning

- The ratio of red counters to blue counters in a box is 1 : 2

  - What fraction of the counters are blue?
  - What fraction of the counters are red?
  - What is the same about the ratio and the fractions? What is different?

- This bar model represents $\frac{2}{5}$



This bar model represents 2 : 5

What is the same and what is different about the bar models?

- Use the diagram to complete the sentences.

The ratio of blue counters to green counters is 2 : _____

The fraction of counters that are blue is $\frac{2}{\square}$

- One third of the chocolates in a box are mint flavoured.

  The rest are strawberry.

  Use diagrams to show that the ratio of mint to strawberry chocolates is 1 : 2

- The bar model shows the ratio 2 : 3 : 4

  - What fraction of the bar is pink?
  - What fraction of the bar is yellow?
  - What fraction of the bar is blue?

- Esther gets $\frac{2}{5}$ of a packet of 30 sweets.

  Huan shares 70 sweets with his friend in the ratio 2 : 5

  How many more sweets does Huan get than Esther?

- Brett opens a box of buttons and counts the different colours.
  - $\frac{1}{2}$ of them are red.
  - $\frac{1}{3}$ them are green.
  - The rest are yellow.

  What is the ratio of red : green : yellow buttons in the box?

Ratio and fractions

Reasoning and problem solving

There are some red and green cubes in a bag.

$\frac{2}{7}$ of the cubes are red.

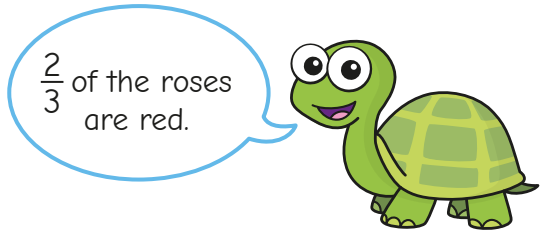
Are the statements true or false?

For every 2 red cubes, there are 7 green cubes.	False
For every 2 red cubes, there are 5 green cubes.	True
For every 5 green cubes, there are 2 red cubes.	True
For every 5 green cubes, there are 7 red cubes.	False

Explain your answers.

Mrs Fisher plants flowers in a flower bed.

For every 2 red roses, she plants 3 white roses.



Is Tiny correct?

Explain your answer.

No

Dani makes 240 ml of squash using cordial and water in the ratio 1 : 3

She adds more water to the cup so there is now 300 ml of squash.

What fraction of the drink is cordial?

$\frac{1}{5}$

Scale drawing

Notes and guidance

In this small step, children apply their understanding of ratio and multiplicative relationships through scale diagrams. Before children begin to draw, it is important to spend time exploring what scale diagrams are by getting them to decide by eye if diagrams are accurately scaled or if the proportion of the dimensions has been changed.

Children become familiar with the language of "Each square represents ..." to explain the relationship between the original image and its scale drawing.

Encourage children to explore different ways of calculating scaled lengths using multiplicative relationships between numbers. For example, if 3 cm represents 9 cm, then to find what 6 cm represents they can either multiply 9 cm by 2 or multiply 6 cm by 3 to find the result, 18 cm.

Once children are confident with this and are able to draw squares and rectangles, they may move on to drawing more complex rectilinear shapes.

Things to look out for

- Children may identify the correct scale of enlargement but still become confused by whether they need to multiply or divide.

Key questions

- How do you know if a diagram is drawn to scale?
- Why might you need to draw a scale diagram?
- If 1 square represents 5 cm, what do _____ squares represent? How do you know?
- If 1 square represents 5 cm, how many squares represent _____ cm? How do you know?
- Is there more than one way of finding the missing value?
- How is a scale like a ratio?

Possible sentence stems

- _____ squares represents _____, so each square represents _____
- Each square represents _____, so _____ squares represent _____ × _____ = _____
- Each square represents _____, so _____ is represented by _____ ÷ _____ = _____ squares.

National Curriculum links

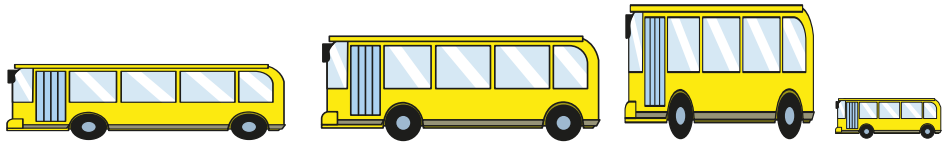
- Solve problems involving similar shapes where the scale factor is known or can be found

Scale drawing

Key learning

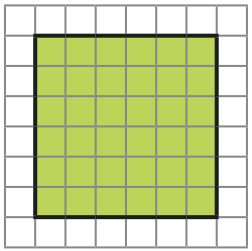
- Here is a picture of a bus.

Which two pictures are scale drawings of the original?



- A square has side lengths of 12 cm.

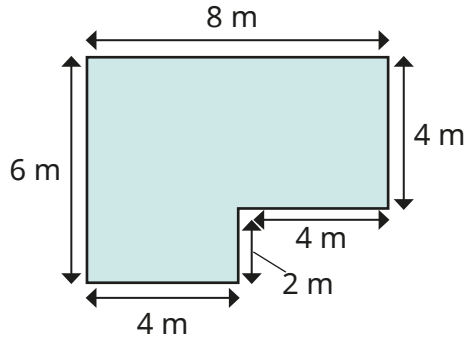
  Scott has drawn a scale diagram of the shape in which the side length of each square in the grid represents 2 cm.



  Use squared paper to draw other scale diagrams using the side length of each square to represent:

  - 3 cm
  - 4 cm
  - 6 cm
  - 12 cm

- This is a plan of a classroom.



  Using squared paper, draw a scale diagram of the classroom if each square on the grid represents 2 m.

- A football pitch measures 48 m by 72 m.

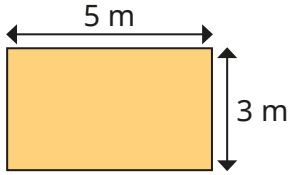
  Using squared paper, draw a scale diagram of the football pitch if each square on the grid represents 8 m.

- On a scale diagram, 4 cm represents 1 m.
  - What does 8 cm represent?
  - What does 40 cm represent?
  - What does 2 cm represent?
  - What does 1 cm represent?
  - What length in centimetres would represent 3 m?

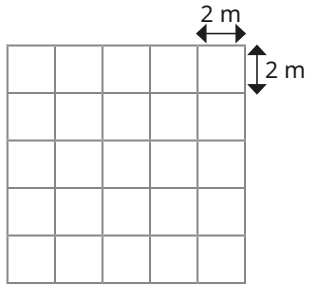
Scale drawing

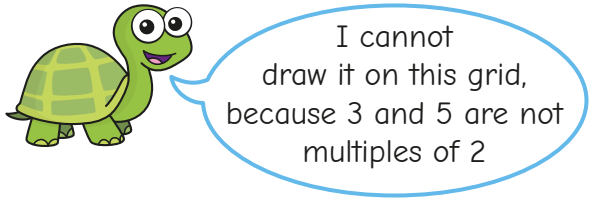
Reasoning and problem solving

Tiny wants to draw a scale diagram of this rectangle.



Each square on the grid represents 2 m.



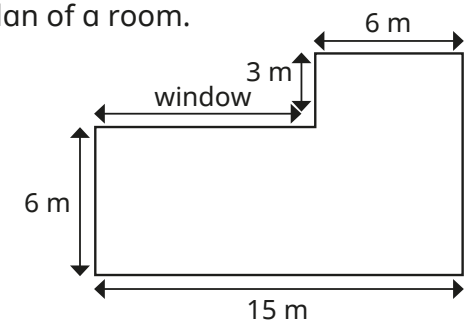


Do you agree with Tiny?

Explain your answer.

No

Here is a plan of a room.



Draw a scale diagram of the room where each square represents 3 m.

What is the actual length of the window?

What is the area, in squares, of the room in the scale diagram?

What is the actual area of the room?

Explain the connection between your answers.

9 m

12 squares

108 m^2

Use scale factors

Notes and guidance

In this small step, children build on the previous step to enlarge shapes and describe enlargements.

Children need to know that one shape is an enlargement of another if all the matching sides are in the same ratio. They can use familiar language such as "3 times as big" before being introduced to the language of scale factors, for example "enlarged by a scale factor of 3". They can then draw the result of an enlargement by a given scale factor. Children also identify the scale factor of an enlargement when presented with both images. Once confident with this, they can explore using inverse operations to find the dimensions of the original shape given the size of the enlargement.

Things to look out for

- Children may not use the scale factor with all the dimensions of the shape.
- Children may use inaccurate measuring when working with shapes with diagonal lines rather than considering the vertical and horizontal distances.

Key questions

- What does "scale factor" mean?
- How do you draw an enlargement of a shape?
- How can you work out the scale factor of enlargement between two shapes?
- If a shape has been enlarged by a scale factor of _____, how can you find the dimensions of the original shape?
- Do you need to multiply or divide to find the missing length? How do you know?

Possible sentence stems

- _____ × _____ = _____
- The shape is _____ times as big, so the scale factor of the enlargement is _____
- If a shape has been enlarged by a scale factor of _____, I need to _____ by _____ to find the original dimensions.

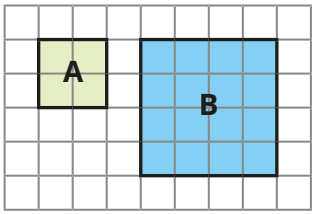
National Curriculum links

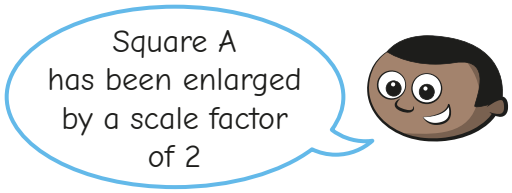
- Solve problems involving similar shapes where the scale factor is known or can be found

Use scale factors

Key learning

- Mo draws a square twice as big as square A and labels it B.

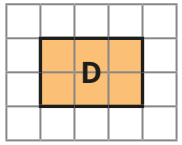




  - Draw a square that is 3 times as big as square A.

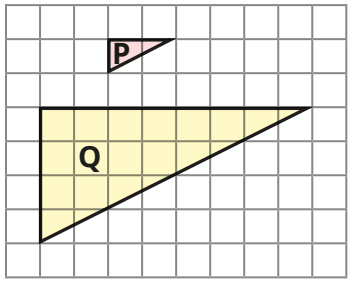
    Label it C.
  - What is the scale factor of enlargement from A to C?

- Use squared paper to complete the enlargements.

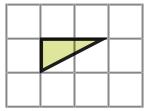


  - Enlarge rectangle D by a scale factor of 2 and label it E.
  - Enlarge rectangle D by a scale factor of 4 and label it F.

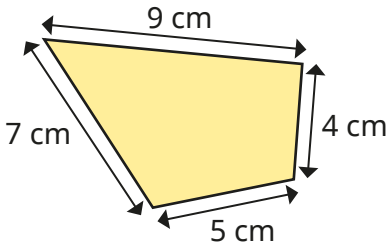
- What is the scale factor of enlargement from P to Q?



- On squared paper, enlarge the triangle by a scale factor of 3



- Here is a quadrilateral.

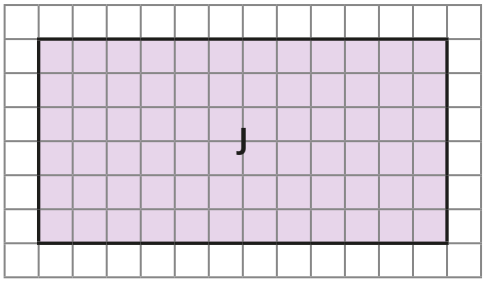


  The shape is enlarged by a scale factor of 7

  What are the lengths of the sides of the enlarged shape?

- A shape is enlarged by a scale factor of 3

  Shape J is the result of the enlargement.

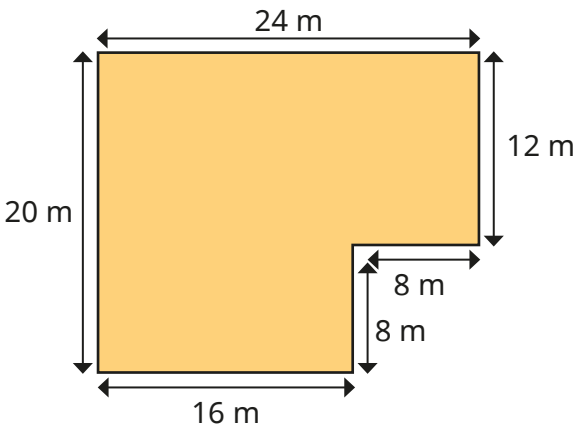


  Draw the original shape.

Use scale factors

Reasoning and problem solving

The shape is the result of an enlargement by a scale factor of 4



What is the perimeter of the enlarged shape?

What is the perimeter of the original shape?

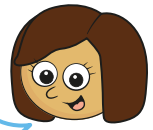
What do you notice?

88 m

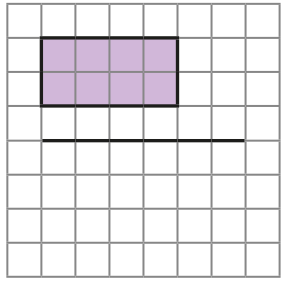
22 m

Kim is enlarging the shape by a scale factor of $1\frac{1}{2}$

I know $\frac{1}{2}$ of 4 is 2, so $1\frac{1}{2}$ multiplied by 4 is 6
The length of the rectangle is 6



Complete the enlargement.



On squared paper, enlarge the shape by a scale factor of $2\frac{1}{2}$

On squared paper, enlarge the shape by a scale factor of $1\frac{1}{4}$

side lengths 6 and 3

side lengths 10 and 5

side lengths 5 and $2\frac{1}{2}$

Similar shapes

Notes and guidance

In this small step, children build on the previous step to explore similar shapes. Similar shapes are defined as shapes where corresponding sides are in the same proportion and the corresponding angles are equal, so if one shape is an enlargement of the other, the two shapes are similar. When testing for similarity, encourage children to work systematically around a shape to ensure that all sides have been enlarged by the same scale factor.

Children can explore the relationship between corresponding angles in the shapes, practising protractor skills learnt in Year 5. Finally, children should apply this understanding to explore similar shapes that are in different orientations, identifying corresponding sides and angles to decide if the shapes are similar.

Things to look out for

- If shapes are in different orientations, children may struggle to identify corresponding sides or just believe the shapes cannot be similar because they do not look the same.
- It is important that children work systematically to ensure all corresponding sides are in the same proportion, rather than just one or two.

Key questions

- What do you think "similar" means?
- What is the scale factor of the enlargement?
- Have all the sides been enlarged by the same amount?
- What are corresponding sides? Can you identify the corresponding sides in these two shapes?
- What do you notice about corresponding angles in similar shapes?
- Does it matter that the shapes are in a different orientation?

Possible sentence stems

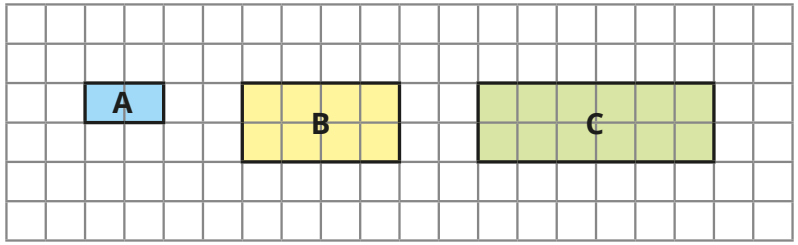
- Each side of the shape is _____ times the size, so the shape has been enlarged by a scale factor of _____. Therefore, the shapes are _____
- I know that the shapes are similar, because the corresponding sides have been enlarged by the same _____, and the corresponding angles are _____

National Curriculum links

- Solve problems involving similar shapes where the scale factor is known or can be found

Similar shapes

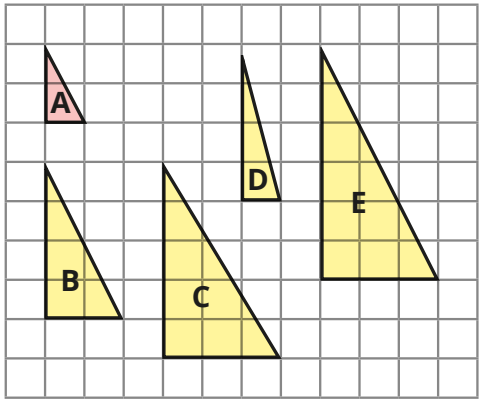
Key learning

- 

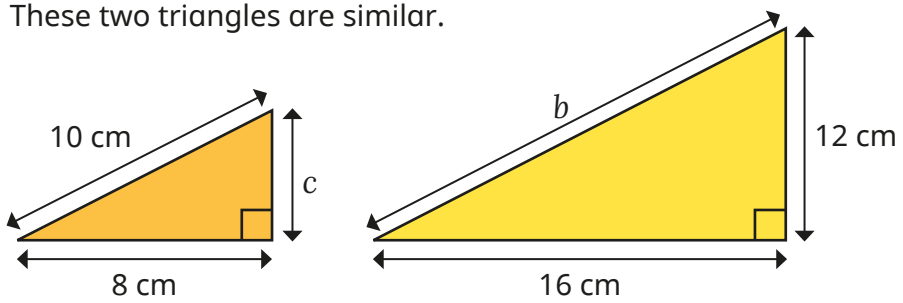
  - Explain why shapes A and B are similar.
  - Explain why shapes A and C are **not** similar.
  - Draw another shape that is similar to A.

  Compare answers with a partner.

- Which of the shapes are similar to shape A?



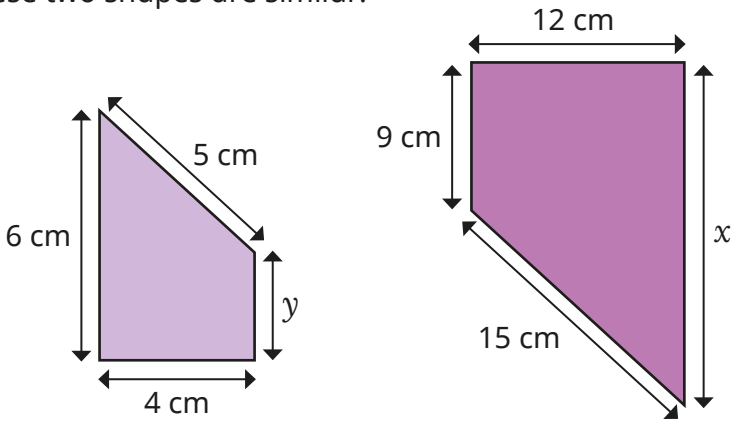
- These two triangles are similar.



  - Find the lengths of b and c.
  - Measure the sizes of all the angles.

  What do you notice?

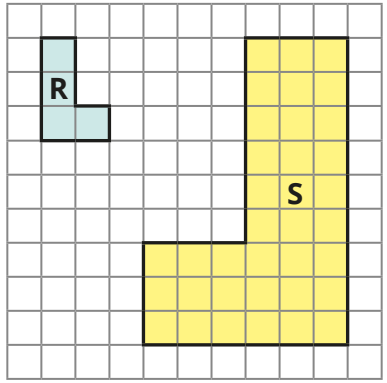
- These two shapes are similar.

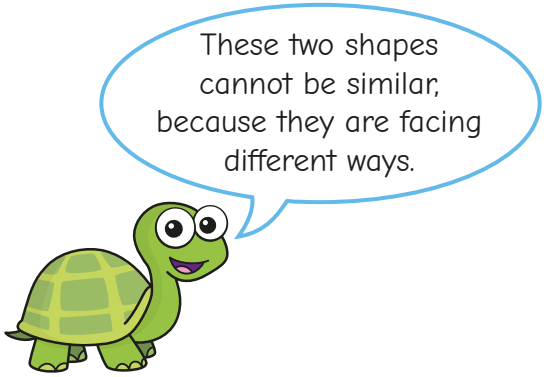


Find the lengths of x and y.

Similar shapes

Reasoning and problem solving





Do you agree with Tiny?

Explain your answer.

No

The Eiffel Tower is 320 m tall and 120 m wide.

Tommy makes a scale model of the Eiffel Tower.

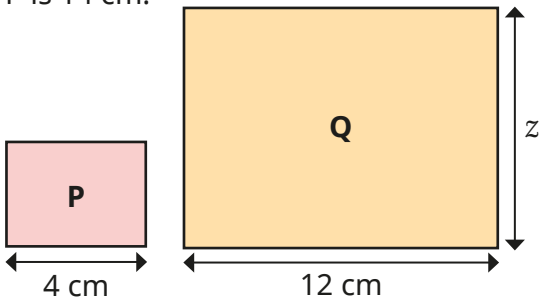
His model is 16 cm tall.

How wide is his model?

6 cm

Rectangles P and Q are similar.

The perimeter of rectangle P is 14 cm.



Work out length z.

$z = 9$ cm

Ratio problems

Notes and guidance

In this small step, children use what they have learnt so far in this block to solve a variety of problems involving ratio.

Children use representations from earlier steps to help them see the multiplicative relationships between ratios. They recognise that when they multiply or divide from one amount to another, they do the same for the other value to keep the ratios equivalent. Children may see that this method is similar to finding equivalent fractions. When using double number lines, children can explore the vertical as well as horizontal multiplicative relationships.

Representing problems using bar models supports the interpretation of word ratio problems. These models can be used for a wide range of question types, such as: "If there are _____ blue/red/total, how many blue/red/total are there?" and "If there are _____ more red than blue, how many blue/red/total are there?"

Things to look out for

- Children may confuse the "total" amount for the value of a missing part.
- Children may use additive rather than multiplicative relationships.

Key questions

- What is the ratio of _____ to _____?
- If there are _____, how many _____ must there be?
- If the total number of _____ is _____, how many _____ must there be?
- If there are _____ more _____ than _____, how many are there in total?
- How can you draw a bar model to solve the problem?
  Which parts of the model do you know?
  How can you work out the remaining parts?

Possible sentence stems

- The ratio of _____ to _____ is _____ : _____
- I know that _____ multiplied/divided by _____ is equal to _____, so to find out how many _____ there are, I need to multiply/divide by _____

National Curriculum links

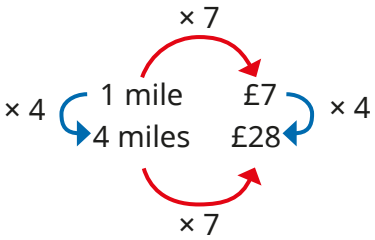
- Solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts

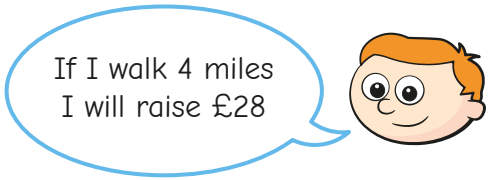
Ratio problems

Key learning

- Ron is doing a sponsored walk for charity.

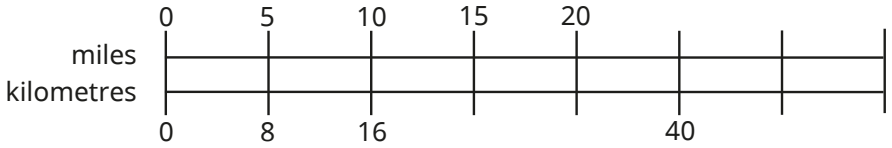
  For every mile he walks, he will raise £7





  - How much will Ron raise if he walks 3 miles?
  - How much will Ron raise if he walks 22 miles?
  - How many miles will Ron need to walk to raise £42?

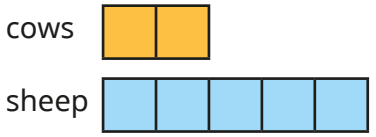
- The double number line shows the relationship between miles and kilometres.
  - Complete the double number line.



  - Complete the statements.

    55 miles = _____ km _____ miles = 96 km

- On a farm, for every 2 cows, there are 5 sheep.



  Use bar models to answer the questions.
  - If there are 4 cows, how many animals are there altogether?
  - If there are 35 animals altogether, how many cows are there?
  - If there are 50 sheep, how many cows are there?
  - If there are 12 cows, how many more sheep are there than cows?

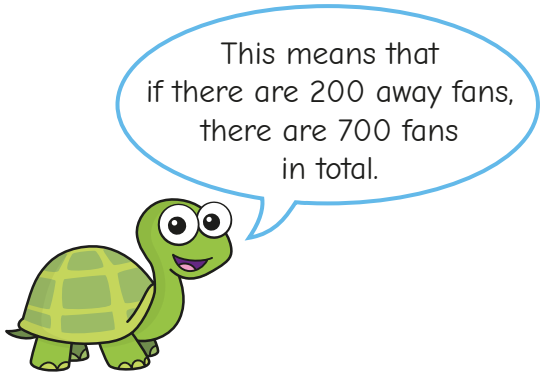
- In a car park, there are 4 blue cars for every 7 red cars.
  - If there are 20 blue cars, how many red cars are there?
  - If there are 28 red cars, how many blue cars are there?
  - If there are 22 cars in total, how many of them are blue?
  - If there are 12 blue cars, how many more red cars are there than blue cars?
  - If there are 30 more red cars than blue cars, how many cars are there in total?

Ratio problems

Reasoning and problem solving

At a football match, the ratio of home fans to away fans is 7 : 2

Home fans	Away fans
7	2
14	4
21	6
28	8

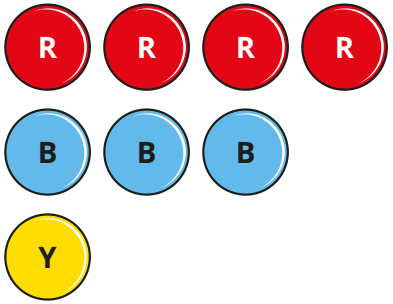


Do you agree with Tiny?

Explain your answer.

No

The ratio of red to blue to yellow counters is 4 : 3 : 1



If there are 148 red counters, how many yellow counters are there?

If there are 50 more blue counters than yellow counters, how many red counters are there?

If there are 608 counters in total, how many of them are red?

How did you work this out?

Compare answers with a partner.

37

100

304

Proportion problems

Notes and guidance

In this small step, children explore different strategies for solving proportion problems.

Building on previous steps, a double number line is a useful representation for these types of problems. Begin by looking at simple one-step problems that involve a single multiplication or division, for example "4 _____ cost _____. What do 12 cost?" or "4 _____ cost _____. What do 2 cost?"

Then move on to two-step problems, where children first need to find the value of 1 _____ through division. Again, seeing this on a double number line helps to show children that both values need to be divided by the same amount to find 1, then both new values can be multiplied by the same amount to find any new value.

Things to look out for

- In one-step proportion problems, children may multiply by the wrong amount or add instead of multiply.
- When using a double number line in two-step proportion problems, children may count the step to zero and divide by the wrong amount.

Key questions

- What is the multiplicative relationship between _____ and _____?
- If 3 _____ cost £_____, how much do 12 _____ cost?
- If 5 _____ cost £_____, how can you work out what 1 _____ costs?
- Once you know what 1 _____ costs, how can you work out what 8 _____ cost?
- How can a double number line help you solve this proportion problem?

Possible sentence stems

- If _____ costs _____, then _____ costs _____
- To get from _____ to _____, I multiply/divide by _____
- To find the cost of 1 _____, I will ...

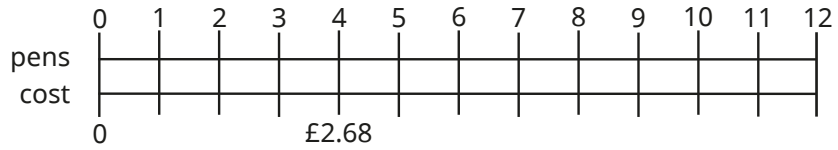
National Curriculum links

- Solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts

Proportion problems

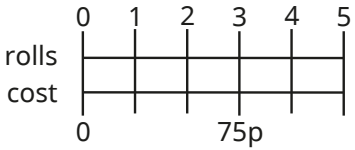
Key learning

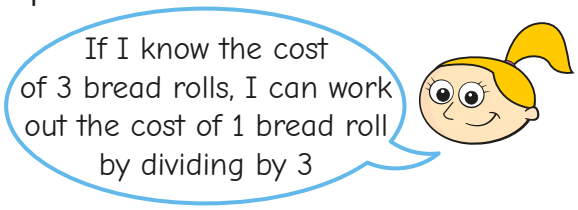
- 4 pens cost £2.68



  - Use the double number line to work out the cost of 12 pens.
  - Use a double number line to help you work out the cost of buying:
    - 36 pens
    - 360 pens
  - Use a double number line to help you work out how many pens can be bought for:
    - £1.34
    - £26.80

- Eva buys 3 bread rolls for 75p.



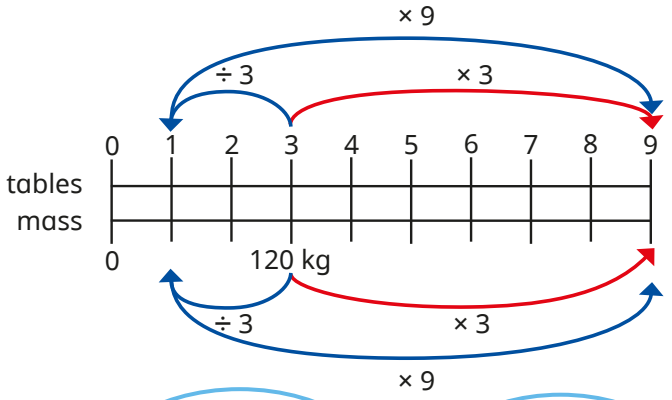


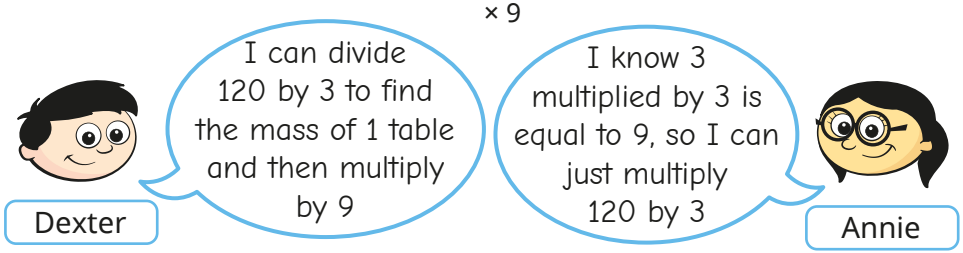
Tell a partner how this will help Eva to find the cost of 5 bread rolls.

What is the cost of 5 bread rolls?

- 3 tables have a total mass of 120 kg.

Dexter and Annie are working out the mass of 9 tables.





Use both methods to find the mass of 9 tables.

Whose method do you prefer?

- A shop sells flour at the price of 54p for 0.3 kg.

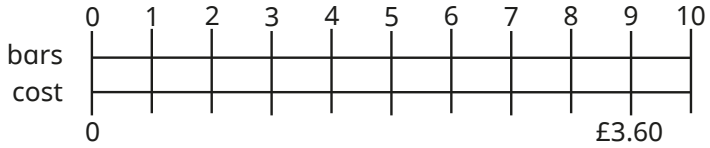
How much would it cost to buy these masses of flour?

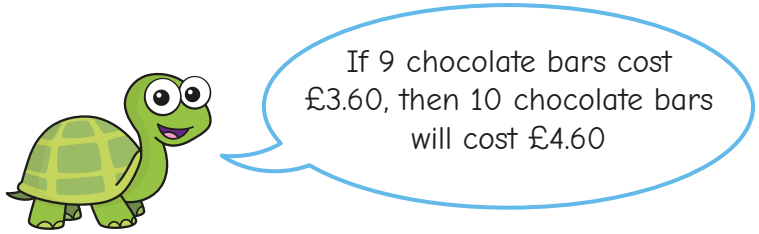
150 g	700 g	2 kg	5.2 kg

Proportion problems

Reasoning and problem solving

The cost of 9 chocolate bars is £3.60





Do you agree with Tiny?

Explain your answer.

No

Tiny has added £1, but each chocolate bar does not cost £1

1 chocolate bar costs £3.60 ÷ 9 = 40p

10 chocolate bars cost 40p × 10 = £4

It costs a company 12p to make 10 marbles.

Marbles are sold in boxes of 500 for £6.50

How much profit does the company make on every box of marbles?

How did you work it out?

50p

A car travelling at a constant speed travels 24 km in 12 minutes.

How far will the car travel in 1 hour?

How long will it take the car to travel 84 km?

How did you work it out?

120 km

42 minutes

Recipes

Notes and guidance

For this small step, children apply their knowledge of ratio and proportion to solving problems involving ingredients for recipes.

As a class, look at a simple list of ingredients for, for example, 4 people and discuss how it could be adapted for 8/2/40 people. After solving simple scaling-up/scaling-down problems, children look at problems with a given amount of a specific ingredient, for example "The recipe needs 100 g of butter. Aisha has 500 g of butter. How much _____ can she make?"

Children can then explore multi-step problems that involve multiplying and dividing quantities of ingredients, for example adjusting the quantities for 4 people to 5 people by dividing each ingredient by 4 and then multiplying by 5

Things to look out for

- Children may only scale one of the ingredients instead of all of them.
- Children may not see efficient methods for two-step problems.
- Children may make errors when they need to convert between units of measure.

Key questions

- How can a double number line help you decide how much of each ingredient you need?
- How many times more people are there? How will this affect the amount of each ingredient needed?
- Do you need to find the amounts needed for one person first? Why or why not?
- What is the greatest number of _____ you can make with _____?
- How does changing the quantities in a recipe link to using scale factors?

Possible sentence stems

- There are _____ times as many people, so I need _____ times as much of each ingredient.
- First, I will find the quantities for 1 person by dividing by _____ and then I will multiply this by _____

National Curriculum links

- Solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts

Recipes

Key learning

- Here are some ingredients for cupcakes.

  Tom wants to make 10 cupcakes.

  Explain to a partner how to work out what ingredients Tom will need.

  How much of each ingredient will Tom need to make the different numbers of cupcakes?

Cupcakes (makes 5)

100 g flour

2 eggs

40 g sugar

15 cupcakes | 20 cupcakes | 25 cupcakes

- Here are some ingredients for soup.

  How much of each ingredient is needed to make soup for the different numbers of people?

Soup (for 6 people)

1 onion

60 g butter

180 g lentils

1.2 litres stock

480 ml tomato juice

2 people | 1 person | 9 people

- Sam is making pancakes.

  She follows a recipe with this list of ingredients.

  She has 1.2 litres of milk and wants to make as many pancakes as she can.

  How many eggs will she need?

Pancakes

120 g plain flour

2 eggs

300 ml milk

- Here are the ingredients for an apple crumble.

  How much of each ingredient is needed to make apple crumble for the different numbers of people?

Apple crumble (5 people)

300 g plain flour

225 g brown sugar

200 g butter

450 g apples

10 people | 12 people

- A baker uses 12 eggs to make 108 muffins.

  How many muffins will 20 eggs make?

  How many different ways can you work it out?

Recipes

Reasoning and problem solving

Here are the ingredients for 10 flapjacks.

Flapjacks (makes 10)
120 g butter
100 g brown sugar
4 tablespoons golden syrup
250 g oats
40 g sultanas

Huan has 180 g butter.

What is the greatest number of flapjacks he can make?

How much of each of the other ingredients will he need?

15

150 g brown sugar

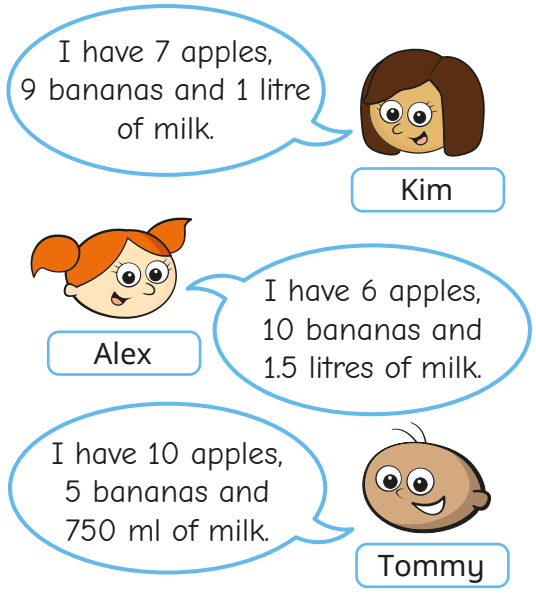
6 tablespoons golden syrup

375 g oats

60 g sultanas

Here are the ingredients for making one smoothie.

Smoothie
2 apples
3 bananas
500 ml milk



Who can make the most smoothies?

Alex